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DETERMINESTATIC TRANSMISSION ERRORS OF INVOLUTE SPUR GEAR BODIES IN MESH APPROPRIATE MODELS OF CONTACT AND BENDING STRESSES USING FINITE ELEMENT ANALYSIS

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ABSTRACT

This thesis investigates the characteristics of an Involute gear system including contact stresses, bending stresses, and the transmission errors of gears in mesh. Gearing is one of the most critical components in mechanical power transmission systems. Transmission error is considered to be one of the main contributors to noise and vibration in a gear set. Transmission error measurement has become popular as an area of research on gears and is possible method for quality control. To estimate transmission error in a gear system, the characteristics of Involute spur gears were analyzed by using the finite element method. The contact stresses were examined using 2-D FEM models. The bending stresses in the tooth root were examined using a 3-D FEM model. Current methods of calculating gear contact stresses use Hertz's equations, which were originally derived for contact between two cylinders. To enable the investigation of contact problems with FEM, the stiffness relationship between the two contact areas is usually established through a spring placed between the two contacting areas. This can be achieved by inserting a contact element placed in between the two areas where contact occurs. The results of the two dimensional FEM analyses from ANSYS are presented.

These stresses were compared with the theoretical values. Both results agree very well. This indicates that the FEM model is accurate. This thesis also considers the variations of the whole gear body stiffness arising from the gear body rotation due to bending deflection, shearing displacement and contact deformation. Many different positions within the meshing cycle were investigated.

Key words: Involute, Transmission, FEM model.

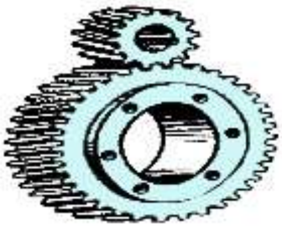
INTRODUCTION

Gearing transmissions have a long history dating back since the time of the first engineering systems. Their practical usage in the present day modern engineering system is enormous. In accordance with a contemporary development of mechanical engineering techniques ever growing requirements and working specifications. Along with modern high speed manufacturing industry development, gears are used widely in many applications ranging from automotive transmission to robot and aerospace engines. Different kinds of metallic gears are currently being manufactured for various industrial purposes. Seventy-four percent of them are spur gears, fifteen percent helical, five percent worm, four percent bevel, and the others are either epicyclical or internal gears. The main purpose of gear mechanisms is to transmit rotation and torque between axes. The gear is a machine element that has intrigued many engineers because of numerous technological problems arises in a complete mesh cycle. If the gears were perfectly rigid and no geometrical errors or modifications were present, the gears would result in a constant speed at the output shaft. The assumption of no friction leads to that the gears would transmit the torque perfectly, which means that a constant torque at the output shaft. No force variations would exist and hence no vibrations and no noise could be created. Of course, in reality, there are geometrical errors, deflections and friction present, and accordingly, gears sometimes create noise to such an extent that it becomes a problem. Transmission error occurs when a traditional non-modified gear drive is operated under assembly errors. Transmission error is the rotation delay between driving and driven gear caused by the disturbances of inevitable random noise factors such as elastic deformation, manufacturing error, alignment error in assembly.

1 TYPE OF GEARS:

SPUR GEAR: Spur gears have their teeth parallel to the axis and are used for transmitting power between two parallel shafts. They are simple in construction, easy to manufacture and cost less. They have highest efficiency and excellent precision rating. They are used in high speed and high load application in all types of trains and a wide range of velocity ratios.

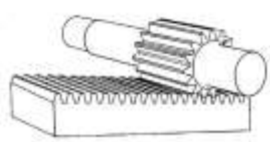
HELICAL GEAR: Helical gears are used for parallel shaft drives. They have teeth inclined to the axis. Hence for the same width, their teeth are longer than spur gears and have higher load carrying capacity. Their contact ratio is higher than spur gears and they operate smoother and quieter than spur gears. Their precision rating is good. They are recommended for very high speeds and loads. Thus, these gears find wide applications in automotive gearboxes.



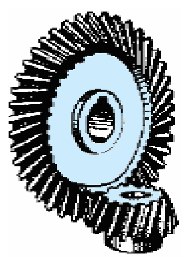
DOUBLE HELICAL GEAR OR HERRINGBONE GEAR: Double helical or Herringbone gears used for transmitting power between two parallel shafts. They have opposing helical teeth with or without a gap depending on the manufacturing adopted. Two axial thrusts oppose each other and nullify. Hence the shaft is free from any axial force. Through their load capacity is very high, manufacturing difficulty makes them costlier than single helical gear. Their applications are limited to high capacity reduction drives like that of cement mills and crushers, one such application is exhibited.

INTERNAL GEAR: Internal gears are used for transmitting power between two parallel shafts. In these gears, annular wheels are having teeth on the inner periphery. This makes the drive very compact. In these drives, the meshing pinion and annular gear are running in the same direction.


RACK AND PINION: Rack is a segment of a gear of infinite diameter. The tooth can be spur or helical. This type of gearing is used for converting rotary motion into translator motion or vise versa.




STRAIGHT BEVEL GEAR: Straight bevel gears are used for transmitting power between intersecting shafts. They can operate under high speeds and high loads. Their precision rating is fair to good. Wide application of the straight bevel drives is in automotive differentials, right angle drives of blenders and conveyors.

SPIRAL BEVEL GEAR: Spiral bevel gears are also used for transmitting power between intersecting shafts. Because of the spiral tooth, the contact length is more and contact ratio is more. They operate smoother than straight bevel gears and have higher load capacity.

WORM GEAR: Worm gearing finds wide application in material handling and transportation machinery, machine tools, automobiles etc. An industrial worm gear box used for converting horizontal to vertical drive.

Transmission Error in Gear : Theoretically, for two gears with perfect involutes and an infinite stiffness, the rotation of the output gear would be a function of the input rotation and the gear ratio. A constant rotation of the input shaft would therefore result in a constant rotation of output shaft. Due to both intended shape modifications and unintended modifications, such as manufacturing errors, gears will be a motion error of the output gear relative to the input gear.The transmission error and mesh stiffness variation is often considered to be the primary excitation of gear noise and a minimization of the transmission error is believed to minimise noise. The definition of transmission error is "the difference between the actual position of the output gear and the position it occupy if the gear drive were perfectly conjugate". The transmission error can be measured statically or dynamically, unloaded or loaded shown in the table.

		Load (Torque)	
		Low	*High*
Speed	*Low*	*Static unloaded*	*Static Loaded*
	High	*Dynamic Unloaded*	*Dynamic Loaded*

Table No. 1: Two Cylinders Contact Stress Simulation

The importance of contact in the mechanics of solids, contact effects are rarely seriously taken into account in formal analysis, because of the extreme This model was built based on the Hertz contact stress theoretical problem. The radii were calculated from the pitch diameters of the pinion and gear and other parameters shown in Table 3.1 and Figure 3.2. The contact stress of this model should represent the contact stress between two gears. The geometry of two half cylinders, must be described. Then the geometry areas were meshed. In contact areas a fine mesh was built. The boundary conditions were applied in this model. The loads also were applied four times as four steps. In each step there are a lot of sub-steps. In each sub-step the number of equilibrium iterations was set. The steel material properties have an elastic Young's modulus of 20,000,000 psi and the Poisson's ratio was 0.40. complexity convoluted. Mechanical problems involving contacts are inherently nonlinear. Usually the loading causes significant changes in stiffness, which results in a structure that is nonlinear.

Number of teeth	25
Normal Module (M)	6 mm
Addendum Modification coefficient	0
Normal Pressure Angle	20 degrees
Face Width (mm)	0.015 M
Addendum (mm)	1.00 M
Dedendum (mm)	1.25 M

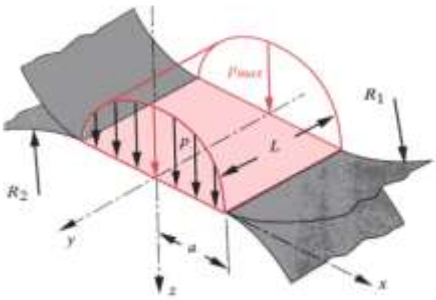
Table No. 2: Specifications of spur gears used

Nonlinear structural behavior arises for a number of reasons, which can be reduced to three main categories:

(1) Geometric Nonlinearities
(2) Material Nonlinearities
(3)Change in Status Nonlinearities (Contact).

3 Hertz Contact Stress Equations

Usually, the current methods of calculating gear contact stresses use Hertz's equations, which were originally derived for contact between two cylinders. Contact stresses between two cylinders were shown in Figure 3.4. An ellipsoidal-prism pressure distribution is generated between the two contact areas.



From Figure 3.4 the width of the contact zone is 2a. If total contact force is F and contact pressure is p(x), there is a formula [5], which shows the relationship between the force F and the pressure p(x):

$$F = 2L\int_0^a p(x)\,dx \quad ,$$

Contact width a = $\sqrt{\dfrac{2F}{\pi L}\dfrac{(1-\upsilon_1^2)/E_1+(1-\upsilon_2^2)/E_2}{1/d_1+1/d_2}}$

The maximum contact stress $P_{max} = \dfrac{2F}{\pi a L}$

$$P_{\max} = \sigma_H = 0.564 \sqrt{\frac{F\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}{\frac{1-\upsilon_1^2}{E_1} + \frac{1-\upsilon_2^2}{E_2}}}$$

F is the load per unit width

R is the radius of cylinder i $R_i = d_i$ sin for the gear teeth $\varphi/2$

φ is pressure angle υ_i is Poisson's ratio for cylinder

E_i *is* Young's modulus for cylinder

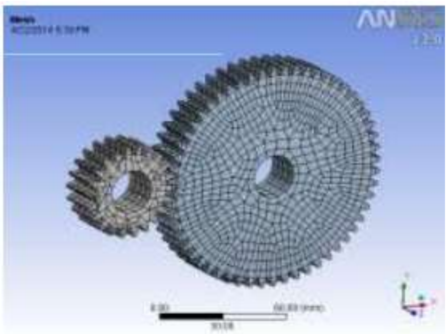
CONTACT AND BENDING STRESS ANALYSIS

The results obtained the present method is an efficient and correct method, which is proposed to calculate the tooth contact stresses of a gear pair. Special techniques of the finite element method were used to solve contact problems in previous chapter. We are using the present method, the tooth contact stresses and the tooth deflections of a pair of spur gears analyzed by ANSYS are given in section 4.4. Since the present method is a general one, it is applicable to many types of gears.

- There is no sliding in the contact zone between the two bodies
- The contact surface is continuous and smooth

The present method ANSYS can solve the contact problem and not be limited by the above two conditions. A two-dimensional and an asymmetric contact model were built, parameter definitions were given and then many points of the involute profile of the pinion and gear were calculated to plot an involute profile using a cylindrical system.

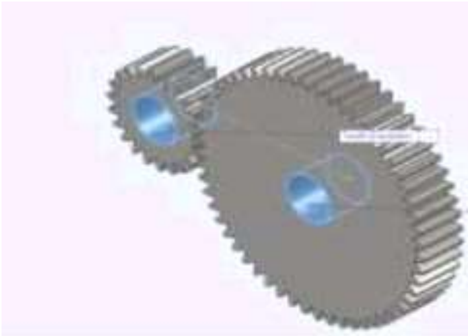
The proper constraints on the nodes were given. The contact pair was inserted between the involute profiles, the external loads were applied on the model from ANSYS "SOLUTION > DEFINE LOAD > FORCE / MOMENT", and finally, ANSYS was run to get the solution.



The bending stress model, evaluating load sharing between meshing gears is compulsory. It is also an important concept for transmission error. It is a complex process when more than one-tooth pair is simultaneously in contact taking into account the composite tooth deflections due to bending, shearing and contact deformation. This section presents a general approach as to how the load is shared between the meshing teeth in spur gear pairs. When the gears are put into mesh, the line tangent to both base circles is defined as the line of action for Involute gears. In one complete tooth mesh circle, the contact starts at points.

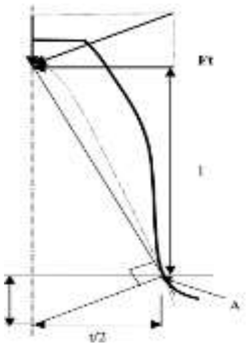
The Lewis Formula

There are several failure mechanisms for spur gears. Bending failure and pitting of the teeth are the two main failure modes in a transmission gearbox. Pitting of the teeth is usually called a surface failure. The bending stresses in a spur gear are another interesting problem. When loads are too large, bending failure will occur. Bending failure in gears is predicted by comparing the calculated bending stress to experimentally determine allowable fatigue values for the given material. This bending stress equation was derived from the Lewis formula. Wilfred Lewis (1892) [5] was the first person to give the formula for bending stress in gear teeth using the bending of a cantilevered beam to simulate



stresses acting on a gear tooth shown in Figure 4.5 are Cross-section = t b* , length = l, load =t F, uniform across the face. For a rectangular section, the area moment of inertia is

$M = F_t l$ and $c = t/2$, stress then is $\sigma = \frac{M}{I/c} = \frac{F_t l(t/2)}{bt^3/12} = \frac{6F_t l}{bt^2}$



Where = b the face width of the gear. For a gear tooth, the maximum stress is expected at point A, which is a tangential point where the parabola curve is tangent to the curve of the tooth root fillet called parabola tangential method. Two points can be found at each side of the tooth root fillet. The stress on the area connecting those two points is thought to be the worst case. The crack will likely start from the point A. From similar triangles.

$$\tan\alpha = \frac{t/2}{x} = \frac{l}{t/2}$$

where $l = \frac{t^2}{4x}$

$$\sigma = \frac{6F_t\frac{t^2}{4x}}{bt^2} = \frac{3F_t}{2bx} = \frac{3F_t p_d}{2bp_d x} = \frac{F_t p_d}{bY}$$

Where p_d = diametral pitch

$Y = \frac{2xp_d}{3}$ = Lewis form factor

Equation [5] in the next page is known as the Lewis equation, and Y is called the Lewis form factor. The Lewis equation considers only static loading and does not take the dynamics of meshing teeth into account. The Lewis form factor is given for various numbers of teeth while assuming a pressure angle of 20^0 and a full depth Involute. The Lewis form factor is dimensionless, and is also independent of tooth size and only a function of shape. The above stress formula must be modified to account for the stress concentration K_c. The concentrated stress on the tooth fillet is taken into account by K_c and a geometry factor Y_j, where $Y_j = Y/K_c$ is introduced. Other modifications are recommended by the AGMA for practical design to account for the variety of conditions that can be encountered in service. The following design equation, developed by Mott (1992) is used.

$$\sigma = \frac{F_t\, p_d K_a K_s K_m}{b Y_j K_v}$$

where K_a = application factor ,

K_s = size factor,

K_m = load distribution factor,

K_v = dynamic factor,

F_t = normal tangential load,

Y_j = Geometry factor.

FINITE ELEMENT MODELS

The maximum stresses on the tensile and compressive sides of the tooth, respectively, are required. The bending stress sensitivity of a gear tooth has been calculated using photo elasticity or relatively coarse FEM meshes. With present computer developments we can make significant improvements for more accurate FEM simulations.

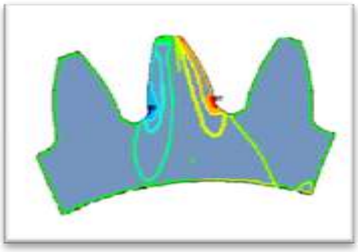


Figure No. 1: Gear tooth bending model

TRANSMISSION ERROR

Static transmission error includes the contact problem of spur gears. A Finite element analysis system has been developed. A two dimensional model can be used instead of a three dimensional model to reduce the total number of the elements and the total number of the nodes in order to save pc memory. Transmission Error the contact problem was sometimes included because the nonlinear problem made the model so critical. It is based on a two dimensional finite element analysis of tooth deflections. Two models were adopted to gain a more accurate static transmission error, for a set of successive positions of the driving gear and driven gear. Two different models of a gear pair have been made to analyze the effects of gear body deformation and the interactions between adjacent loaded teeth. Results are from each of the two models ‘good values. The noise generated by a pair of gears is mainly concerned to the gear transmission error. The main source of patent innervations in gearboxes is created by the meshing process. Researchers generally assume that transmission error and the variation in gear mesh stiffnesses are responsible for the noise generated by the gearbox. The term transmission error is used to describe the deviation between the theoretical and actual relative angular rotations between a pinion and a gear. Its characteristics depend on the instantaneous positions of the meshing tooth pairs. These positions result from tooth deflections and manufacturing errors. The transmission error is mainly caused by:

a) Tooth geometry errors: including profile, spacing and run out errors from the manufacturing process;
b) Elastic deformation: local contact deformation from each meshing tooth pair and the deflections of teeth because of bending and shearing due to the transmitted load.
c) Imperfect mounting: geometric errors in alignment, which may be introduced by static and dynamic elastic deflections in the supporting bearings and shafts.

In the pinion reference frame: the local cylindrical system number 12 was created by definition in ANSYS. By constraining the all nodes on the pinion in radius and rotating θg with the gear having a torque input load the model was built. In this case, 0 = θp and θg is in the opposite direction to that resulting from forward motion of θp changing the TE result to positive as seen by equation

$$TE = \theta_g - (z)\theta_p$$

The Contact Stress Analysis

The contact stress analyses were to gain an understanding of the modeling and solution difficulties in contact problems and examine the contact stresses in the gears. Contact between two cylinders was modeled. To reduce time, only half cylinders were meshed in the model as shown in Figure 6.1(a). The fine meshed rectangular shaped elements were generated near contact areas shown as 6.1 (b). The dimensions of the elements are based on the half width of the contact area. The contact conditions are sensitive to the geometry of the contacting surfaces, which means that the finite element mesh near the contact zone needs to be highly refined. It is recommended not to have a fine mesh everywhere in the model to reduce the computational requirements.

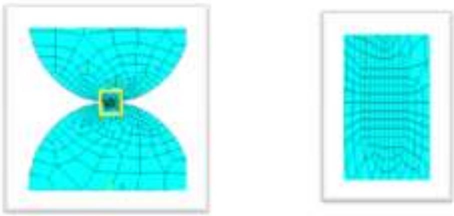
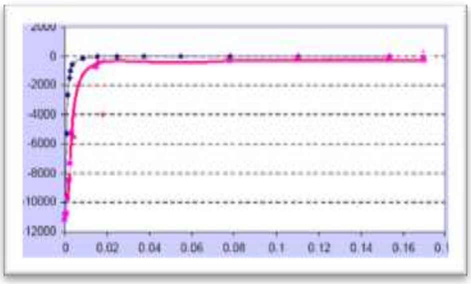


Figure No. 2: Rectangular shaped

the stresses σx σy as a function of depth y along a radius of the cylinder. The depth is normalized to the half-width *a* of the contact patch. This plot provides a dimensionless picture of the stress distribution on the centerline under an



ellipsoidal contact. Note all the stresses have diminished to <10% of P_{max} within $y = 5a$.

$$\sigma_y = \frac{y}{\pi}[a\beta - x\alpha]F_{\max}$$

$$\tau_{xy} = \frac{1}{\pi} y^2 \alpha F_{\max}$$

Where the factors α and β are given by

$$\alpha = \frac{\pi}{k_1} \frac{1 - \sqrt{\frac{k_2}{k_1}}}{\sqrt{\frac{k_2}{k_2}} \sqrt{2\sqrt{\frac{k_2}{k_1}} + (\frac{k_1 + k_2 - 4a^2}{k_1})}}$$

$$\beta = \frac{\pi}{k_1} \frac{1 + \sqrt{\frac{k_2}{k_1}}}{\sqrt{\frac{k_2}{k_2}} \sqrt{2\sqrt{\frac{k_2}{k_1}} + (\frac{k_1 + k_2 - 4a^2}{k_1})}}$$

$k_1 = (a + x)^2 = y^2$

$k_2 = (a - x)^2 + y^2$

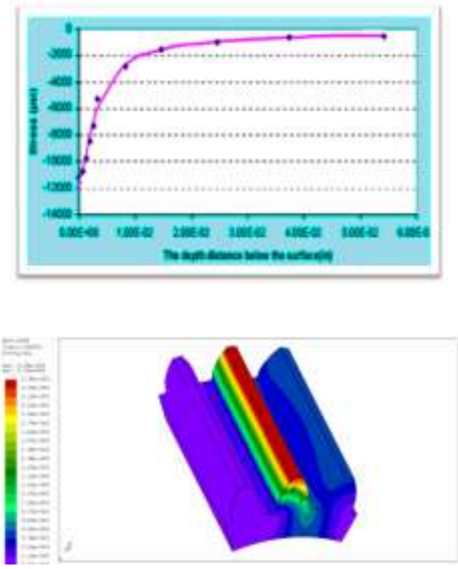


Figure No. 3: Comparison between calculated values and ANSYS values

A comparison of the tooth root stresses obtained in the three dimensional model and in the two dimensional model using ANSYS. Analyses of gears with different numbers of teeth are carried out. The number of gear teeth is 25. The meshing spur gear has pitch radius of 60 mm and a pressure angle of 20°. The gear face width, b = 1.5 in (38.1mm). The transmitted load is 2500 N.

If the number of teeth is change from 25 to 31 with the parameters kept the same.

$$\sigma = \frac{F_t\, p_d K_a K_s K_m}{b Y_j K_v}$$

$$= \frac{542.011 * 6.124 * 1 * 1 * 1.15}{1.5 * 0.37 * 0.8 * 100}$$

= 85.972MPa

$$\sigma = \frac{F_t\, p_d K_a K_s K_m}{b Y_j K_v}$$

$$= \frac{542.011 * 6.096 * 1.2 * 1.2 * 1.15}{1.5 * 0.37 * 0.8 * 100}$$

= 123.233MPa

If the number of teeth is changed from 31 to 25 and the other parameters were kept the same.

$$\sigma = \frac{F_t\, p_d K_a K_s K_m}{b Y_j\, K_v}$$

$$= \frac{542.011 * 4.21 * 1.2 * 1.2 * 1.15}{1.5 * 0.37 * 0.8}$$

= 85.107MPa

NO. Of teeth	Stress 3D (2D) (AGMA)	Stress(AGMA)	Difference 3D (2D)
25	95.802 (91.129)	85.872	4.39% (0.69%)
28	109.21 (106.86)	102.78	6.26% (3.97%)
31	123.34 (116.86)	124.805	8.39% (2.69%)

Table No. 3: Von Mises Stress Of 3D and 2D FEM bending model

CONCLUSION

In the thesis, Finite element modeling of the contact between two cylinders was analyzed in point. The finite element method with special techniques, such as the incremental technique of applying the external load in the input file, the deformation of the stiffness matrix, and the introduction of the contact element were used. It was observed that initial loading using displacements as inputs was helpful in decreasing numerical instabilities. Effective methods to estimate the tooth contact stress by the two-dimensional and the root bending stresses by the three-dimensional and two-dimensional finite element method are proposed. Find out the accuracy of the current method for the bending stresses, both three dimensional and two dimensional models were progress. The results with the different numbers of teeth were used in the comparison. It is found that Lewis formula and Hertz equation is used for quick stress calculation for gear, where as the AGMA standards and FEM is used for detailed gear stress calculation for a pair of Involute spur gear. The errors are less as compare to previous work, shown in the above Table. So those Finite element Analysis models are good enough for stress analysis.

REFERENCES

[1] Gatcombe, E.K., Prowell, R.W., 1960, "Rocket motor gear tooth analysis", (Hertzian contact stresses and times) Trans. ASMA, J. Engg. Industry, page no. 238-250

[2] Mark, W. D., 1978, "Analysis of the vibratory excitation of gear system: Basic theory", J. Acoust. Soc. Am., 63, page no. 1409-1430.

[3] Errichello, R., 1979, "State-of-art review: gear dynamics", Trans. ASME, J. Mech. Des., 101(3), page no. 368-372.

[4] Coy, J. J., Chao, C. H. S., 1982, "A method of selecting grid size to account for Hertz deformation in finite element analysis of spur gears", Trans. ASME, J. Mech. Design 104 page no. 759-766.

[5] Kohler, H., Regan, R., 1985, "The derivation of gear transmission error from pitch error records", Proc. Instn Mech. Engrs, Part C, Journal of Mechanical Engineering Science, 199(C3), page no. 195-201.

[6] Tsay, C.B., 1988, "Helicalgears with involute shapedteeth: Geometry, computer simulation, tooth contact analysis, and stress analysis", Trans, J. Mechanisms, Transmissions, and Automation in Design, page no. 121-135.

[7] Umezawa, K., 1988, "Recent Trends in Gearing Technology", JSME International Journal Series III Vol.31, No. 2, page no. 357-362.

[8] Ozguven, H. N., Houser. D. R., 1988,"Mathematical models used in gear dynamics", Sound Vibr., 121,page no. 383-411.

[9] Kubo, A., et al., 1991, "Estimation of transmission error of cylindrical Involute gears by tooth contact pattern", JSME int.J., Ser. III, 34(2), page no. 252-259.

[10] Blarasin, A., Guagliano, M., Vergani, L., 1997, "Fatigue Crack Growth Predictions in Specimens Similar to Spur Gear Teeth", Fatigue Fract. Eng. Mater. Struct., 20, No. 8,page no. 1171-1182.

[11] Emmanuel Rigaud, Ecole Centrale De Lyon, et al., 1999 Modelling And Analysis Of Static Transmission Error Effect Of Wheel Body Deformation And Interactions Between Adjacent Loaded Teeth, Laboratoire de Tribologie et Dynamique des Systèmes, page no. 435-443.

[12] Sirichai S., 1999, "Torsional Properties of Spur Gear inMesh using nonlinear Finite Element Analysis", Ph.D. Thesis, Curtin University of Technology.

[13] Tappeta, R. V., 2001, "Interactive Multiobjective Optimization Design Strategy for Decision Based Design", Journal of Mechanical Design. Vol, 123 page no..205-215.

[14] David, L. G., Handschuh F. R., 2001, "Consideration of Moving Tooth Load in Gear Crack Propagation Predictions", Transations of the ASME vol.123 page no. 118-124.